Assignment 5

1. Show that $[a,b]$ can be expressed as the intersection of countable open intervals. It shows that countable intersection of open sets may not be open.

2. Can we write $[a,b)$, $a,b \in \mathbb{R}$, as the countable union of open intervals? The same question for countable union of closed intervals, countable intersection of open intervals and countable intersection of closed intervals.

3. Show that the only open and closed set in $\mathbb{R}$ is the empty set and $\mathbb{R}$ itself. Suggestion: Let E be such a set consider the endpoints of an interval contained in E.

4. Show that every open set in $\mathbb{R}$ can be written as a countable union of disjoint open intervals. Suggestion: Introduce an equivalence relation $x \sim y$ if x and y belongs to the same open interval in the open set and observe that there are at most countable many such intervals.

5. (a) Let $f,g \in C[a,b]$ and $D = \{(x,y) : \ f(x) < g(x),\ x \in (a,b)\}$ and $F = \{(x,y) : f(x) \le g(x),\ x \in [a,b]\}$. Show that D is open and F is closed in $\mathbb{R}^2$.

   (b) Let $h,j \in C[c,d]$ and $G = \{(x,y) : \ h(y) < x < j(y),\ f(x) < y < g(x),\ (x,y) \in (a,b) \times (c,d)\}$. Show that G is an open set in $\mathbb{R}^2$.

6. Let $A = \big\{f \in C[-1,1] : \ f(0) = 2,\ f(-1) = -5\big\}$. Show that A is closed in $C[-1,1]$.